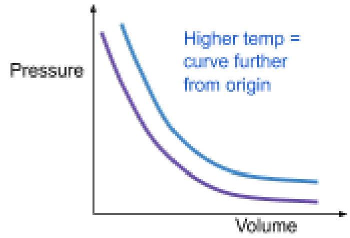
CAIE Physics A-level

Topic 15: Ideal Gases

Notes

15 - Ideal Gases



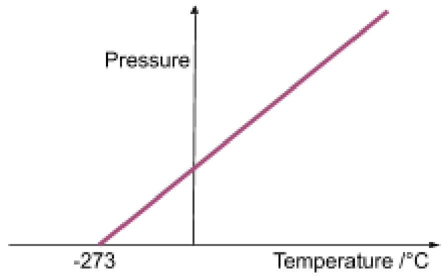
15.1 - The Mole

The amount of substance is a base quantity denoting the number of particles in a substance. The base unit of the amount of substance is the mole, and the number of moles is labelled *n*. One mole is defined as the number of atoms in 12g of Carbon-12. This is just a very large number, called the **Avogadro Number,** $N_A = 6.02 \times 10^{23}$ per mole.

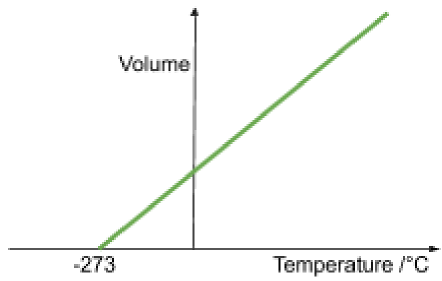
The total number of particles in a substance is then $N = nN_A$.

15.2 - Equation of State

The **gas laws** describe the experimental relationship between pressure (p), volume (V), and temperature (T) for a fixed mass of gas:



1. **Boyle's Law** -When temperature is constant, pressure and volume are inversely proportional $pV = K$



2. **Charles' Law** -When pressure is constant, volume is directly proportional to absolute temperature $V/T = K$

3. **The Pressure Law** -When volume is constant, pressure is directly proportional to absolute temperature $p/T = K$

An **ideal gas** follows the gas laws perfectly, meaning that there is no other interaction other than **perfectly elastic collisions** between the gas molecules. This shows that no intermolecular forces act between molecules.

You can combine all the experimental gas laws into one to get $\frac{pV}{T} = K$ where the constant K is dependent on the amount of gas used measured in **moles**, therefore you can rewrite the above equation to get $\frac{pV}{T} = nR$, where n is the number of moles of gas, and R is the molar gas constant (8.31 J mol^{-1} K^{-1}). You can rearrange this further to get the ideal gas equation:

$$pV = nRT = NkT$$

Where p is the pressure of the gas, V is the volume, n is the number of moles, T is the temperature in Kelvin and R is the molar gas constant (8.31 J mol-1 K-1). N is the number of particles and k is the Boltzmann constant given by $k = R/N_A$.

The above equation is also known as the **equation of state for an ideal gas**, and below is an example question using it:

Find the pressure of 16 g of helium at 25°C, occupying a volume of 4.0×10^{-4} m^3.

The relative atomic mass of helium is 4 (meaning that 1 mole of helium has a mass of 4 g).

Firstly, calculate the temperature in Kelvin.

$25 + 273 = 298$ K

Next, calculate the number of moles of gas.

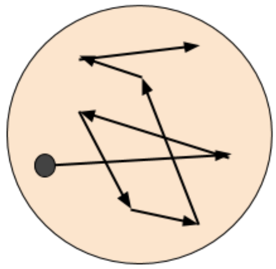
$\frac{16}{4}$ = 4 moles

Finally, rearrange the ideal gas equation so pressure is the subject and substitute your values to calculate pressure.

$$p = \frac{nRT}{V} \qquad = \frac{4 \times 8.31 \times 298}{4.0 \times 10^{-4}} = 2.5 \times 10^{7} \text{ Pa}$$

15.3 - Kinetic Theory of Gases

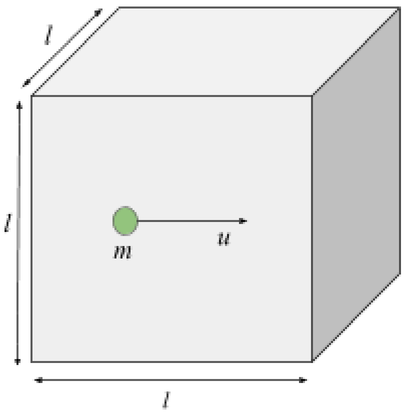
Brownian motion is the random motion of larger particles in a fluid caused by **collisions** with surrounding particles, and can be observed through looking at smoke particles under a microscope. Brownian motion contributed to the evidence for the existence of molecules and their movement.



The **kinetic theory model** equation relates several features of a fixed mass of gas, including its pressure, volume and mean kinetic energy. There are several underlying **assumptions**, which lead to the derivation of this equation. These assumptions and the derivation are outlined below. Note that the assumptions describe an ideal gas.

Assumptions -

- **No intermolecular forces** act between the molecules
- The **duration of collisions is negligible** in comparison to time between collisions
- The motion of molecules is **random**, and they experience **perfectly elastic collisions**
- The motion of the molecules follows **Newton's laws**
- The molecules **move in straight lines** between collisions



Derivation -

1. First, you must consider a cube with side lengths l, full of gas molecules. One of these molecules, has a mass m and is travelling towards the right-most wall of the container, with a velocity u. Assuming it collides with this wall elastically, its **change in momentum** is $mu - (-mu) = 2mu$.

2. Before this molecule can collide with this wall again it must travel a distance of $2l$. Therefore the time between collisions is t, where $t = \frac{2l}{u}$.

3. Using these two bits of information we can find the **impulse**, which is the rate of change of momentum of the molecule. As impulse is equal to the **force** exerted, we can find **pressure** by dividing our value of impulse by the area of one wall: l^2 .

$$F = \frac{2mu}{\frac{2l}{u}} = \frac{mu^2}{l} \qquad\qquad P = \frac{\frac{mu^2}{l}}{l^2} = \frac{mu^2}{l^3} = \frac{mu^2}{V}$$

As shown, the above equation can be further simplified because l^3 is equal to the cube's volume (V).

4. The molecule we have considered is one of many in the cube, the total pressure of the gas will be the sum of all the individual pressures caused by each molecule.

$$P = \frac{m((u_1)^2 + (u_2)^2 + ... + (u_n)^2)}{V}$$

5. Instead of considering all these speeds separately, we can define a quantity known as **mean square speed**, which is exactly what it sounds like, the mean of the square speeds of the gas molecules. This quantity is known as $\overline{u^2}$, and we multiply it by N, the number of particles in the gas, to get an estimate of the sum of the molecules' speeds.

$$P = \frac{Nm\overline{u^2}}{V}$$

6. The last step is to consider all the directions the molecules will be moving in. Currently we have only considered one dimension, however the particles will be moving in all 3 dimensions. Using pythagoras' theorem we can work out the speed the molecules will be travelling at:

$$\overline{c^2} = \overline{u^2} + \overline{v^2} + \overline{w^2}$$

Where u, v, and w are the components of the molecule's velocity in the x, y and z directions.
As the motion of the particles is random we can assume the mean square speed in each direction is the same.

$$\overline{u^2} = \overline{v^2} = \overline{w^2} \quad \therefore \quad \overline{c^2} = 3\overline{u^2}$$

The last thing to do now it put this into our equation and rearrange:

$$pV = \tfrac{1}{3}Nm\overline{c^2} \text{ or } pV = \tfrac{1}{3}Nm < c^2 >$$

As $\overline{c^2}$ and $< c^2 >$ are equivalent

The ideal gas equation and kinetic theory model equation are both modelled on ideal gases and are equal to pV (the product of pressure and volume), meaning they can be equated:

$$pV = \tfrac{1}{3}Nm < c^2 > \qquad pV = nRT$$

$$\tfrac{1}{3}Nm < c^2 > \ = nRT$$

$$\tfrac{1}{3}Nm < c^2 > \ = \tfrac{N}{N_A}RT \quad \text{As } n = \tfrac{N}{N_A} \text{ (from section 1.3)}$$

$$\tfrac{1}{3}m < c^2 > \ = \tfrac{R}{N_A}T \quad \text{As the Ns cancel out}$$

$$\tfrac{1}{3}m < c^2 > \ = kT \quad \text{As } k = \tfrac{R}{N_A}$$

$$\tfrac{1}{2}m < c^2 > \ = \tfrac{3}{2}kT$$

The equation on the left is the equation for the **translational** kinetic energy of a molecule in the gas, and 3k/2 is a constant, meaning that the (average) translational kinetic energy of a molecule in a gas and its temperature are **directly proportional**.

Below is an example question using the above equations.

A bottle contains 128 g of oxygen at a temperature of 330 K. Find the sum of the kinetic energies of all the oxygen molecules. Relative atomic mass of oxygen = 32.

Firstly, find the number of moles of gas, then multiply this by the avogadro constant to find the number of molecules.

Number of moles = $\frac{mass}{atomic\ mass} = \frac{128}{32} = 4$

Number of molecules = $4 \times 6.02 \times 10^{23} = 2.408 \times 10^{24}$

Then, use $\frac{3}{2}kT$ (derived above) to find the kinetic energy of one molecule and multiply this by the number of molecules:

Kinetic energy of a single molecule = $\frac{3}{2} \times 1.38 \times 10^{-23} \times 330 = 6.831 \times 10^{-21}$

Sum of kinetic energies = $6.831 \times 10^{-21} \times 2.408 \times 10^{24} = 16450$ J